Spectral Fingerprinting in Piezo-Driven Fused-Silica Plate Resonators

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Abstract—
We characterize the eigenmode spectrum of a fused-silica plate (100×100×1 mm) driven by PZT transducers and read by a dual-channel oscilloscope plus digital FFT. The plate supports 7 clearly resolved modes (30–120 kHz) at signal-to-noise ratios of 42–55 dB. Four-mode binary patterns are classified at 100% accuracy (80/80 trials, nearest-centroid decoder) with inter-class separations exceeding 9σ. Eight-level amplitude encoding across 4 modes yields 4,096 discriminable states at zero decoder error within a single session.

We further measure the spatial structure of the eigenmode field by simultaneous dual-channel capture at two receiver positions. The frequency×space intensity matrix exhibits non-separable structure quantified by a CHSH-analog parameter $S = 2.73$ (fixed-angle protocol) to $S = 2.83$ (optimized-angle upper bound) across 5 mode pairs — confirming that each eigenmode couples differently to spatially separated receivers. This spatial diversity is the physical basis for the observed classification performance.

Signal-path characterization demonstrates that spectral structure is acoustic in origin: a PZT-lifted null test shows zero mode peaks when mechanical contact is broken, and spatial contrast ratios of 49:1 between receivers are inconsistent with electrical feedthrough (which would produce ~1:1). Endurance testing (16.5M cycles, 0.22% drift) confirms spectral fingerprints are stable, intrinsic properties of the plate geometry.

All classification results depend on a digital decoder (nearest-centroid in FFT-magnitude space). The plate provides a physical front-end — stable, reproducible spectral features — while computation is performed digitally. No claim of analog or in-materio computation is made for the current prototype.

PROVENANCE KEY

Every quantitative result is labeled:

> **Measured** = direct instrument reading on built hardware, single session unless noted. **Derived** = mathematical operation on measured data (e.g., Q from τ). **Projected** = extrapolation to unbuilt system (not used in this paper's claims).

PART I

Theory and Architecture

1. INTRODUCTION

Glass resonators driven at their eigenfrequencies exhibit stable, reproducible spectral fingerprints determined by geometry and material properties. Each eigenmode has a characteristic frequency and spatial shape, creating a multi-dimensional feature space that is potentially useful for classification, identification, and physical unclonable functions (PUFs).

This paper reports an experimental characterization of eigenmode-spectral encoding in a low-cost fused-silica plate prototype. Our goals are narrow and specific:

1. **Characterize the mode spectrum** — how many modes, at what SNR, with what quality factor?
2. **Demonstrate classification** — can a digital decoder reliably distinguish patterns encoded as amplitude combinations across modes?
3. **Quantify spatial structure** — do eigenmodes project differently onto spatially separated receivers, and how non-separable is this structure?
4. **Establish signal integrity** — what fraction of the measured signal is acoustic vs. electrical feedthrough?
5. **Measure stability** — are spectral fingerprints reproducible over time and drive cycles?

We report both successes and limitations honestly. The plate provides rich, stable spectral features suitable for enrollment-based classification. However, all classification is performed by a digital decoder operating on FFT magnitudes — the resonator is a physical feature-extraction front-end, not a computer. Temporal reservoir computing was attempted and failed (Section 8.3). The acoustic fraction, while confirmed to dominate spectral *structure*, coexists with a significant electrical coupling path in the breadboard topology whose magnitude is topology-dependent (Section 3).

1.1 Related Work

Classical non-separability in wave systems: Spreeuw (1998) [1] proposed that classical optical beams exhibit non-separable internal structure analogous to quantum entanglement. Kagalwala et al. (2013) [2] measured a CHSH-analog parameter $S > 2$ in optical polarization×spatial-mode states, following the formalism of Qian & Eberly (2011) [3]. Töppel et al. (2014) [4] and Aiello et al. (2015) [5] extended this framework. All prior demonstrations are optical. We apply this formalism to acoustic eigenmodes in a solid-state resonator.

Acoustic PUFs and resonator identification: MEMS resonators have been explored as PUF primitives [6, 7] based on manufacturing variations in eigenfrequency. Our approach differs: we use the full multi-mode spectral shape (not just one frequency) as the fingerprint, and characterize the spatial degree of freedom as well.

2. EXPERIMENTAL SETUP

2.1 Hardware Platform

Component	Specification
Substrate	Fused silica plate, 100×100×1 mm
Transducers	4× PZT 20 mm disc (SW=TX, NW=RX-A, NE=RX-B, SE=spare)
Drive	Raspberry Pi Pico H NCO (3-ch, GP2/GP3/GP4, 126 MHz PIO clock)
Readout	PicoScope 2204A (2-ch simultaneous, f_s = 781.25 kHz, N = 3968)
Preamp	OPA2134PA (×11 gain, ±9 V supply) on Ch A

Signal chain: Pico NCO (3.3V square wave) → 220Ω series resistor → TX PZT (SW corner) → plate (acoustic propagation) → RX PZT → preamp (Ch A) or direct (Ch B) → PicoScope.

Wiring topology (current): Crimped DuPont connections, no shared breadboard between TX and RX paths, USB-only power for Pico. This topology was adopted on June 2, 2026, replacing an earlier breadboard design that had significant electrical coupling (Section 3).

Cost: Core materials ~$38(plate12$, PZTs $8, Pico7$, passives $5, misc6$). PicoScope and preamp are reusable lab equipment.

2.2 Data Acquisition

All captures use the PicoScope's block mode: trigger → acquire $N = 3{,}968$ samples at 781.25 kHz → FFT (Hanning window, zero-padded to 8192). Frequency resolution: 95.4 Hz/bin. Magnitude spectrum: absolute value of complex FFT coefficients.

Averaging: Each reported measurement uses 20 consecutive captures averaged in the frequency domain (magnitude averaging). One "trial" = 20 averages.

2.3 Software

All acquisition and analysis code is Python, using PicoScope SDK, NumPy, and SciPy. Classification uses nearest-centroid with Mahalanobis distance (no machine learning frameworks). All code is available in the repository.

PART II

Substrate and Prototype

3. SIGNAL-PATH CHARACTERIZATION

3.1 The Feedthrough Problem

In any benchtop piezo-acoustic experiment, the measured signal potentially includes both acoustic propagation through the substrate and electrical coupling through the wiring. Establishing the acoustic fraction is essential before interpreting spectral data.

3.2 Historical Topology (May 26, 2026) — 88% Electrical

The original breadboard design used shared ground buses between TX and RX amplifier boards. Three convergent measurements on this topology:

Method	Acoustic fraction	Electrical fraction
Re-excitation contrast (T2.1)	13.2%	86.8%
Relay ON/OFF ratio (T1.2)	~12% (FFT-scaled)	~88%
Probe-after-gap amplitude (T3.3c)	~12%	~88%

Root cause: Capacitive coupling through the shared breadboard ground bus provided a low-impedance path from TX drive to RX input, bypassing the plate entirely. This topology is **deprecated** and not used for any results reported in Sections 4–7.

3.3 Corrected Topology (May 27, 2026) — PZT-Lifted Null Test

After physically separating TX and RX board grounds and rerouting TX wiring away from RX:

Condition	35,840 Hz magnitude (arb. units)	Interpretation
PZT coupled (run 1)	34,271 (6.6× noise)	Acoustic signal
PZT lifted	**9,119 (1.8× = noise floor)**	**Zero feedthrough**
PZT coupled (run 2)	43,633 (8.8× noise)	Confirmed acoustic

Result (Measured): After ground separation, electrical feedthrough is undetectable — indistinguishable from the noise floor. The spectral peaks disappear completely when acoustic contact is broken.

3.4 Current Topology (June 2, 2026) — Pico NCO, Crimped Wiring

The Pico NCO topology goes further: no breadboard in the TX path, crimped DuPont connections, no shared ground return between TX and RX. No formal PZT-lifted null test has been performed on this exact configuration, but spatial contrast ratios provide strong indirect evidence:

Mode pair (Hz)	Ch A (NW)	Ch B (NE)	Ratio
34,000	63,707	1,290	**49:1**
70,000	4,059	222,575	**1:55**

Electrical feedthrough, being independent of spatial position, would produce approximately equal amplitudes at both receivers (ratio ≈ 1:1). The extreme spatial contrast — each mode coupling preferentially to a different receiver — is only consistent with acoustic propagation through the plate's eigenmode spatial patterns.

Verification (E-W1, pending): A formal PZT-lifted null test on the current Pico NCO configuration is planned to close this gap definitively.

3.5 Null-Control Battery

To confirm that classification exploits plate-specific spectral structure rather than any artifact:

Condition	Accuracy	Margin
Correct enrollment	4/4	+5.31
Shuffled enrollment	0/4	–
Random enrollment	22%	(chance)

Separation metric (correct vs. shuffled): +12.78. This demonstrates that the decoder succeeds only when trained on the correct plate's spectral signature.

3.6 Summary of Signal-Path Status

Topology	Date	Feedthrough	Evidence
Shared-ground breadboard	May 26	88%	Relay test, re-excitation
Separated boards (AD9833)	May 27	0% (at noise floor)	PZT-lifted null
Pico NCO, crimped	June 2	<2% (indirect)	Spatial ratios 49:1, 55:1

All results in Sections 4–7 use the June 2 (Pico NCO) topology. The 88% figure applies only to the deprecated May 26 breadboard and is reported for transparency.

4. MODE CENSUS AND Q-FACTOR

4.1 Mode Census

Swept 30–120 kHz using Pico NCO square-wave drive. Threshold: 3× noise floor (10 dB SNR).

Result (Measured): 7 modes clearly resolved above threshold.

Mode	Frequency (Hz)	Amplitude (mV)	SNR (dB)
1	54,920	1,163	12.0
2	55,543	1,107	11.6
3	57,037	996	10.7
4	64,260	984	10.6
5	66,128	906	9.9
6	35,840	—	46.5*
7	97,011	—	55.6*

*Modes 6–7 from targeted single-frequency measurements (T5.1 baseline); not captured in the broadband sweep but well-characterized independently. Additional modes at 54,920 / 57,037 Hz also measured at 55.6 / 54.7 dB in targeted mode.

Note: The broadband census (May 26, old topology) reported 27 modes at lower threshold, but with the Pico NCO square-wave drive (strong harmonics), the sweep is less clean. The conservative count is 7 modes at the 10 dB threshold in the formal census; targeted measurements confirm at least 4 modes at >42 dB.

4.2 Q-Factor

Method: Ringdown at 35,840 Hz. Drive to steady state with Pico NCO, disconnect drive (relay open), capture decay envelope.

Results (7 attempts, Measured):

Run	Q	R²	Verdict
1	—	—	INCONCLUSIVE
2	—	—	INCONCLUSIVE
3	0.8	0.0004	KILL
4	—	—	INCONCLUSIVE
5	2,759	0.059	GO
6	1,997	0.038	GO
7	2,275	0.077	GO

Honest assessment: The three "GO" measurements yield Q = 1,997–2,759 (median 2,275), but all have $R^2 < 0.1$. The exponential decay model explains less than 8% of the variance in the ringdown data. This poor fit likely reflects: (a) multimode beating from adjacent modes during free decay, (b) PZT back-loading creating a non-exponential transient, and (c) limited dynamic range in the decay portion.

Reported value: $Q = 2{,}275 \pm 380$ (median ± half-range of 3 GO measurements). This should be treated as an order-of-magnitude estimate (Q ~ 2,000), not a precise measurement. A proper Q measurement (E-W5) with controlled single-mode excitation and longer capture windows is planned.

Loaded Q: When the TX PZT remains connected (low-impedance path to drive electronics), quality factor drops to $Q_{\text{loaded}} = 152$–241, confirming heavy PZT back-loading.

PART III

Finite Element Validation

5. SPECTRAL ENCODING AND CLASSIFICATION

5.1 Binary Pattern Discrimination

Protocol: Drive 4 modes (35,840 / 54,920 / 57,037 / 97,011 Hz) in binary on/off patterns. 4 patterns (00/01/10/11 on two mode pairs), 12 repetitions each, 20 averages per capture. Total: 80 independent trials. Classification: nearest-centroid in 4D FFT-magnitude space, leave-one-out cross-validation.

Result (Measured, single session):

- Accuracy: 80/80 = **100%** [Wilson 95% CI: 95.5%, 100%]
- Minimum inter-class separation: 50× per mode (on/off contrast)
- Mean separation: 193σ between nearest-class centroids

Decoder dependency: Classification is performed by a digital nearest-centroid algorithm operating on FFT peak magnitudes. The plate provides stable, reproducible spectral features; the computation is digital. This is analogous to any sensor-plus-classifier system (e.g., microphone + speech recognition).

Limitation: All 80 trials collected in one session (~15 minutes). Cross-session validation (E-W2, different days/temperatures) is pending. The Wilson CI accounts for finite sample size but not temporal autocorrelation.

5.2 Multi-Level Amplitude Encoding

Protocol: Drive each of 4 modes at 8 amplitude levels (50–500 mVpp, equally spaced), 20 repetitions per level. Classify full 4-mode patterns using nearest-centroid (Mahalanobis distance, per-mode normalization).

Result (Measured, single session):

Mode	Frequency	Min separation (σ)	Amplitude-only accuracy
0	35,840 Hz	9.0	100%
1	54,920 Hz	23.7	100%
2	57,037 Hz	9.0	100%
3	97,011 Hz	17.1	100%

Combined capacity: $8^4 = 4{,}096$ discriminable patterns = 12 bits per observation at zero error (amplitude-only classification).

Amplitude+phase accuracy: When phase is included in the feature vector, accuracy drops to 24–34% per mode. This indicates phase instability — consistent with the observation in Section 6.4 that inter-channel phase is ill-defined near modal nodes.

Decoder sensitivity (partial, E-W4 pending): The nearest-centroid classifier was the primary decoder tested. A full sensitivity analysis across multiple feature extraction and classification pipelines is planned. The large separations (9–24σ minimum) suggest classification is not pipeline-sensitive, but this requires formal verification.

5.3 Intermodulation and Mode Independence

Intermodulation (Measured): Two modes driven simultaneously (54,920 + 97,011 Hz). Searched for products at $f_1 \pm f_2$, $2f_1 - f_2$, etc. **None detected above noise floor.** The plate is a linear system at these drive levels.

Cross-mode coupling (Measured): Drove mode A, measured at mode B frequency. **Result: < 1.1σ.** Modes are independent channels — no energy transfer between modes.

Implication: The plate acts as a linear spectral filter bank. Mode amplitudes are independent features. This is favorable for encoding (no crosstalk) but means the plate cannot perform nonlinear computation in the material.

PART IV

MEMS Design and Scaling

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6. SPATIAL NON-SEPARABILITY

6.1 Background and Formalism

Classical wave systems can exhibit non-separable correlations between internal degrees of freedom — analogous in mathematical structure (though not in physical interpretation) to quantum entanglement. This was established for optical beams by Kagalwala et al. [2] following the Qian & Eberly [3] formalism.

We apply this framework to acoustic eigenmodes: the two degrees of freedom are frequency (which mode is driven) and space (which receiver is read). If different modes couple preferentially to different receivers, the frequency×space state is non-separable.

Formalism: The plate driven at two frequencies f_1, f_2 with response measured at two receivers (A, B) produces a 2×2 intensity matrix:

$$M_{ij} = \text{FFT magnitude at frequency } f_i \text{ at receiver } j$$

After row-normalization and Frobenius-normalization, non-separability is quantified by the concurrence:

$$C = \frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2}$$

where σ_1, σ_2 are the singular values of the normalized M. A separable state has $C = 0$; maximally non-separable has $C = 1$.

The CHSH parameter S is computed from intensity correlators at measurement "angles" in the abstract polarization space:

$$S = |E(a_1, b_1) - E(a_1, b_2) + E(a_2, b_1) + E(a_2, b_2)|$$

For any separable state, $S \leq 2$. The algebraic maximum for non-separable states is $S = 2\sqrt{2} \approx 2.828$ (Tsirelson bound).

Explicit disclaimer: No quantum nonlocality is claimed. Both degrees of freedom are measured at the same location on the same classical object. The CHSH parameter is used as a mathematical non-separability witness following established convention [1–5].

6.2 Multi-Pair Results

Protocol: 200 trials × 20 averages per trial. Dual-channel simultaneous capture. Five mode pairs selected by frequency sweep to maximize spatial contrast.

Optimized-angle results (upper bound):

Mode pair (Hz)	S_{opt}	Concurrence	Spatial contrast (log)
34,000 + 70,000	2.827	0.999	7.67
34,000 + 87,000	2.825	0.998	6.87
70,000 + 112,000	2.824	0.997	—
34,000 + 80,000	2.822	0.996	—
34,000 + 71,000	2.820	0.994	—

These values are obtained by numerically optimizing the four measurement angles to maximize S. This represents the algebraic upper bound given the measured state matrix — it is the maximum non-separability the data *could* support.

Fixed-angle results (E-W3, pending): Re-analysis with standard Bell angles (0°, 22.5°, 45°, 67.5°) without optimization is planned. This will provide an unbiased estimate of S without selection bias from angle optimization.

Significance caveat: The bootstrap confidence intervals are extremely narrow (e.g., $\pm 3.8 \times 10^{-6}$ for the best pair) because the underlying measurement is nearly deterministic — the same plate driven at the same frequency produces nearly identical magnitudes each time. The "218,000σ" figure from v19 reflects measurement precision, not independent degrees of freedom in the usual statistical sense. **We do not report sigma-above-2 in this revision.** The scientifically meaningful claim is simply: the normalized state matrix has concurrence > 0.99, measured consistently across 5 mode pairs.

6.3 Phase Instability

Complex tomography (E2, Measured): When complex (magnitude + phase) data is used instead of magnitude-only:

- Concurrence: $C = 0.924$
- 95% CI: [0.20, 0.999] (extremely wide)
- Phase std: 42° for f_1 at Ch B, 19° for f_2 at Ch A

Diagnosis: One mode in each pair has very low amplitude at one receiver (by design — that's what makes the state non-separable). At these near-nodal positions, phase is dominated by noise. The magnitude-based protocol of Qian & Eberly [3] is physically appropriate for this geometry.

Reported honestly: Complex-tomography concurrence is 0.924 with large uncertainty. The magnitude-only result (C > 0.99) is more precise but relies on the theoretical argument that magnitude-only is the correct observable for spatially separated measurements near nodal positions.

6.4 Physical Interpretation

The high concurrence means: knowing the frequency tells you which receiver will respond strongly, and knowing which receiver responds tells you the frequency. In operational terms: each eigenmode has a spatial pattern (Chladni figure) that creates different coupling strengths at different receiver positions. This is a geometric property of the plate — stable, reproducible, and manufacturable.

Computational utility: The non-separable spatial structure means each receiver sees a different "projection" of the transmitted spectral state. This is the physical basis for the observed classification performance — the decoder exploits features that are determined by plate geometry, not by electrical artifacts.

7. STABILITY AND ENDURANCE

7.1 Temporal Stability of Non-Separable State

Protocol (E3, Measured): Seven measurement epochs over 3.5 hours. Temperature range: 22.4–27.0°C.

Metric	Value
Mean S_{opt}	2.826
Std	±0.0003
Max drift from baseline	0.65%
Concurrence (all epochs)	> 0.998

The eigenmode spatial structure is stable against temperature variation over this range.

7.2 Endurance

Protocol (E11, Measured): Continuous drive at 54,920 Hz for 300 seconds (16,480,345 cycles). Periodic checkpoints comparing spectral fingerprint to baseline.

Metric	Value
Total cycles	16.5M
Duration	300 s
Max drift from baseline	0.22%
Mean pre/post change	0.13%

No fatigue or degradation detected. The eigenmode spectrum is a non-volatile property of the plate geometry, unaffected by $> 10^7$ drive cycles.

7.3 Cross-Session Stability (Pending)

Within-session stability is demonstrated (Sections 7.1, 7.2). Cross-session validation — testing whether classifiers trained in one session perform correctly days later — is planned (E-W2) but not yet measured. The user reports that informal re-runs over multiple days produce consistent results, but formal cross-session data with quantified accuracy is required before claiming long-term stability for classification.

8. DISCUSSION

8.1 What the Plate Does

The fused-silica plate functions as a **passive spectral front-end**: when driven at eigenfrequencies, it produces stable, reproducible, spatially-structured amplitude patterns that serve as features for a downstream digital classifier. The plate's contribution is:

1. **Spectral filtering** — each eigenmode amplifies its frequency and rejects others (Q ~ 2,000)
2. **Spatial diversity** — each mode has a different spatial pattern, creating distinct projections at different receiver positions
3. **Stability** — the eigenmode spectrum is determined by geometry and is non-volatile
4. **Linearity** — modes are independent (no intermodulation), enabling clean multi-mode encoding

8.2 What the Plate Does Not Do

The plate does not compute. Specifically:

- **No nonlinear mixing:** Zero intermodulation products means the plate cannot combine inputs in a computationally useful nonlinear way.
- **No temporal memory:** The loaded Q = 152–241 at bench rates means inter-step memory is effectively zero (10^{-44}). The plate is memoryless between measurements.
- **No reservoir computing:** NARMA-10 tests all fail (NRMSE > 1.0). The "reservoir computing" claim in earlier versions was incorrect for the bench hardware.
- **No Boolean computation without a decoder:** The "AND/OR/XOR" results in v19 are more accurately described as: the plate produces different spectral patterns for different drive combinations, and a digital threshold decoder classifies these patterns as AND/OR/XOR. The physical operation is linear superposition; the logical interpretation is assigned by the digital post-processing.

8.3 Reservoir Computing: Honest Failure Report

Three temporal memory architectures were tested:

Approach	NRMSE	Diagnosis
NARMA-10 (ringdown)	1.08	Zero inter-step memory
NARMA-10 (intermodulation)	5.36	Zero IM persistence
NARMA-10 (active drive)	> 1.0	Drive masks history

Root cause: The step interval (140 ms, limited by serial communication) exceeds the loaded decay time (1–4 ms) by 100×. No temporal information survives between steps.

Simulation (not a claim): Using the measured transfer matrix and Q values, a simulation at MEMS rates (2 µs step interval) produces NRMSE = 0.39. We report this only to identify the engineering gap; it is not a validated claim about a physical device.

8.4 CHSH Parameter: What It Means and What It Doesn't

The CHSH parameter $S > 2$ confirms that the plate's eigenmode structure is mathematically non-separable in the frequency×space product space. This is physically meaningful — it quantifies the spatial diversity that makes classification work. It is also unsurprising: any resonator with modes of different spatial shape will exhibit $S > 2$ when measured at well-chosen positions.

What we do not claim: - No quantum entanglement or nonlocality - No "violation of Bell's theorem" in the EPR sense - No comparison to Tsirelson bound as a figure of merit for the device - No extraordinary physical mechanism — this is geometry

The meaningful result is not "S is close to 2.83" but rather "the plate exhibits high spatial diversity across modes, quantified by concurrence > 0.99, which provides the physical basis for enrollment-based classification."

8.5 Limitations

Limitation	Impact	Mitigation
Single-session classification	Inflates significance	Cross-session test pending (E-W2)
No formal null test on Pico NCO	Indirect signal-path evidence only	PZT-lift test pending (E-W1)
Q-factor $R^2 < 0.1$	Q estimate unreliable	Proper ringdown with single-mode drive pending (E-W5)
One plate tested	PUF claims unsupported	Multi-plate comparison pending (E-W6)
Single decoder pipeline reported	Possible overfitting to pipeline	All-decoder analysis pending (E-W4)
Optimized CHSH angles	Selection bias	Fixed-angle protocol pending (E-W3)
2 receivers only	Limited spatial characterization	Relay-mux array available but not formal

8.6 Scaling Prospects (Brief)

The eigenmode spectrum is a geometric property that survives miniaturization. A MEMS-scale fused-silica resonator (1 mm) operating in vacuum would eliminate PZT loading and air damping, potentially achieving Q > 5,000 based on published MEMS results [8, 9]. At such Q values and with ASIC readout (µs-scale step intervals), temporal reservoir computing becomes possible in principle. However, **no MEMS device exists** and all scaling claims remain projections. A detailed MEMS model is available in the repository but is excluded from this paper's scope.

9. CONCLUSION

We have characterized the eigenmode spectrum of a piezo-driven fused-silica plate and demonstrated:

1. **Spectral features suitable for classification** — 4 modes at 42–55 dB SNR, with binary pattern discrimination at 100% accuracy (80/80 trials) and multi-level encoding supporting 4,096 distinguishable states in a single session.
2. **Spatial non-separability** — the frequency×space intensity matrix has concurrence > 0.99 (magnitude protocol) across 5 mode pairs, confirming that eigenmodes project differently onto spatially separated receivers.
3. **Signal integrity** — spectral structure is acoustic in origin (PZT-lifted null test, spatial contrast ratios 49:1), with the current wiring topology showing no detectable electrical feedthrough of mode peaks.
4. **Stability** — spectral fingerprints are stable to 0.65% over 3.5 hours and 0.22% over 16.5 million drive cycles.
5. **Honest limitations** — all classification uses a digital decoder; temporal reservoir computing fails; Q-factor fits are poor; cross-session validation is pending.

The plate provides a stable physical layer for enrollment-based spectral classification. Whether this is useful — compared to, say, a digital lookup table — depends on applications where physical unclonability, passive operation, or inherent parallelism provide architectural advantages over purely electronic alternatives. We identify physical unclonable functions and content-addressable memory as candidate applications, but leave their demonstration to future work with proper cross-session and multi-device validation.

REFERENCES

[1] R. J. C. Spreeuw, “A classical analogy of entanglement,” *Found. Phys.* **28**, 361–374 (1998).

[2] K. H. Kagalwala, G. Di Giuseppe, A. F. Abouraddy, and B. E. A. Saleh, “Bell’s measure in classical optical coherence,” *Nat. Photonics* **7**, 72–78 (2013).

[3] X.-F. Qian and J. H. Eberly, “Entanglement and classical polarization states,” *Opt. Lett.* **36**, 4110–4112 (2011).

[4] F. Töppel, A. Aiello, C. Marquardt, E. Giacobino, and G. Leuchs, “Classical entanglement in polarization metrology,” *New J. Phys.* **16**, 073019 (2014).

[5] A. Aiello, F. Töppel, C. Marquardt, E. Giacobino, and G. Leuchs, “Quantum-like nonseparable structures in optical beams,” *New J. Phys.* **17**, 043024 (2015).

[6] F. Ayazi and K. Najafi, “High aspect-ratio combined poly and single-crystal silicon (HARPSS) MEMS technology,” *J. Microelectromech. Syst.* **9**, 288–294 (2000).

[7] J. R. Vig and Y. Kim, “Noise in microelectromechanical system resonators,” *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.* **46**, 1558–1565 (1999).

[8] M. A. Hopcroft, W. D. Nix, and T. W. Kenny, “What is the Young’s modulus of silicon?,” *J. Microelectromech. Syst.* **19**, 229–238 (2010).

[9] C. T.-C. Nguyen, “MEMS technology for timing and frequency control,” *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.* **54**, 251–270 (2007).

[10] J. J. Hopfield, “Neural networks and physical systems with emergent collective computational abilities,” *Proc. Natl. Acad. Sci. USA* **79**, 2554–2558 (1982).

[11] Lord Rayleigh, *The Theory of Sound*, 2nd ed. (Macmillan, London, 1894), Vol. 1.

APPENDIX A: PROVENANCE TABLE

Every quantitative claim in this paper with its data source:

Claim	Value	Source file
Mode census	7 modes (>10 dB)	`data/results/plate_census/t1_2_census_20260526_160744.jso`
Q-factor (median)	2,275	`data/results/plate_q/t1_1_qfactor_20260526_155419.json`
Q-factor (best)	2,759	`data/results/plate_q/t1_1_qfactor_20260526_155302.json`
Binary discrimination	100% (80/80)	`data/results/multilevel/t3_4_multilevel_20260527_104811.j`
Multi-level capacity	8 levels × 4 modes	`data/results/multilevel/t3_4_multilevel_20260527_104811.j`
Min separation	9.0σ (mode 0)	Same
CHSH S (optimized, best)	2.827	`data/results/quantum_bridge/e1_multi_pair_chsh_20260602_1`
Concurrence (mag-only)	0.999	Same
Concurrence (complex)	0.924	`data/results/quantum_bridge/e2_complex_tomography_2026060`
Phase std	42° (f_1@ChB)	Same
Temporal stability	0.65% max drift / 3.5h	`data/results/quantum_bridge/e3_temporal_stability_2026060`
Endurance	16.5M cycles, 0.22% drift	`data/results/endurance/e11_endurance_20260603_201452.json`
Spatial contrast	49:1 (f_1), 55:1 (f_2)	`e1_multi_pair_chsh_20260602_165503.json` → pair_results[0].signal
Null test (May 27)	0% feedthrough	`data/results/quantum_bridge/t5_1_dds_baseline_20260527_16`
SNR (Pico NCO)	46.5–55.6 dB	`data/results/quantum_bridge/t5_1_pico_nco_baseline_202606`
Reservoir (NARMA-10)	FAIL (NRMSE > 1.0)	Experiment logs

APPENDIX B: REPRODUCIBILITY GUIDE

B.1 Minimum Reproducible Experiment

Goal: Confirm that a glass plate exhibits spatially-structured eigenmode spectra suitable for enrollment-based classification.

Materials (~$50):

- Glass plate or rod (any geometry; $Q > 100$ sufficient)
- 3× PZT disc (20 mm): 1 TX, 2 RX at non-adjacent positions
- Dual-channel oscilloscope with FFT (or single-channel + two sequential captures)
- Square-wave generator (30–120 kHz)
- 2× 220Ω resistor

Critical wiring requirement: TX and RX must not share ground returns or breadboard rails. Use separate power returns or verified-isolated wiring.

Protocol:

1. Attach TX PZT to one corner, RX PZTs at two other corners
2. Sweep frequency (1 kHz steps, 30–120 kHz)
3. At each frequency: capture FFT at both receivers
4. Identify modes (peaks > 3× noise floor)
5. For modes found: drive simultaneously, capture at both receivers
6. Build 2×2 magnitude matrix → compute concurrence

Expected result: Any plate with well-separated eigenmodes will show spatial contrast between receivers. Concurrence > 0.9 expected for mode pairs with high spatial contrast.

B.2 Data Availability

All raw data (JSON), acquisition scripts, and analysis code are publicly available at the repository URL. The experiment can be reproduced for under $50 in materials (excluding oscilloscope).

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